All We See are 3D Projective Shapes

**VIC PATRANGENARU**  
Professor  
Department of Statistics  
Florida State University

**Abstract:** It is known that for \( k \geq 5 \), a manifold of projective shapes of \( k \)-ads in 3D has a structure of a \( 3k - 15 \) dimensional Lie group that is equivariantly embedded in an Euclidean space, based on the fact that the quaternion multiplication on the unit sphere \( S^3 \subset \mathbb{R}^4 \) induces a natural group multiplication on \( (\mathbb{R}P^3)^{k-5} \). This projective quaternion multiplication structure is explored in testing for mean change in populations of matched pairs of 3D projective shapes, as well as in testing for mean projective shape difference of two independent populations on this Lie group.

On the other hand, emulating human vision, in absence of occlusions, the 3D projective shape of a spatial \( k \)-ad can be recovered from a stereo pair of images, thus allowing to test for mean 3D projective shape difference of two spatial scenes. The methodology for testing mean projective shape change in matched pairs was successfully applied to glaucoma detection in animal models by Crane and Patrangenaru (2011); their results, nevertheless, are not applicable to two independent population of images, and in addition, the sample sizes are usually not equal in this case. The goal of this talk is to present the state of the art in (i) within and between matching of features on two 3D scenes from their bilateral digital images and 3D projective shape reconstruction from bilateral views of 3D scenes based on matched features, (ii) designing a two sample tests for 3D means on the manifold of projective shapes of \( k \)-ads in 3D, exploring the non-commutative Lie group structure of this manifold, and, (iii) applying these to a statistical analysis of 3D scenes from digital camera images.

**Biosketch:** Dr. Patrangenaru, who holds PhDs in Mathematics (Haifa Univ.) as well as Statistics (Indiana Univ.) is widely considered to be one of the top researchers working in the interface of Differential Geometry and Statistics. His particular areas of research include Object Data Analysis, Nonparametric Statistics on Manifolds, Applications of Statistics in Medical Imaging, Proteomics, Phylogenetics and Pattern Recognition. His work is generally of interest to mathematicians, statisticians, computer scientists, biologists (especially those who work in bioinformatics or evolutionary biology), and anyone who needs to work with data that has novel, non-traditional features. His most recent book "Nonparametric Statistics on Manifolds and Their Applications" will be published later this year by CRC Press.

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